

## 7 Displacement of Reference Points

Models describing the displacements of reference points due to various effects are provided. These models relate the regularized position  $X_R(t)$  of the reference points (see Chapter 4) to their instantaneous positions. Two kinds of displacements are distinguished: those that affect the reference markers on the crust and those that affect the reference points of the instruments, which are technique-dependent. The first category includes (a) deformations of the solid Earth due to ocean tidal loading as well as those due to the body tides arising from the direct effect of the external tide generating potential and centrifugal perturbations caused by Earth rotation variations, including the pole tide, (b) atmospheric loading. The second category presently only includes the thermal deformation of a VLBI antenna.

### 7.1 Displacement of Reference Markers on the Crust

#### 7.1.1 Local Site Displacement due to Ocean Loading

Ocean tides cause a temporal variation of the ocean mass distribution and the associated load on the crust and produce time-varying deformations of the Earth. The modeling of the associated site displacement is dealt with in this section. The displacement model does not include the translation of the solid Earth that counterbalances the motion of the oceans' center of mass. This convention follows Farrell (1972).

##### Ocean Loading

Three-dimensional site displacements due to ocean tide loading are computed using the following scheme. Let  $\Delta c$  denote a displacement component (radial, west, south) at a particular site and time  $t$ . Let  $W$  denote the tide generating potential (*e.g.* Hartmann and Wenzel, 1995; Tamura, 1987; Cartwright and Tayler, 1971; Cartwright and Edden, 1973),

$$W = g \sum_j K_j P_2^{m_j}(\cos \psi) \cos(\omega_j t + \chi_j + m_j \lambda), \quad (1)$$

where only degree two harmonics are retained. The symbols designate colatitude  $\psi$ , longitude  $\lambda$ , tidal angular velocity  $\omega_j$ , amplitude  $K_j$  and the astronomical argument  $\chi_j$  at  $t = 0^h$ . Spherical harmonic order  $m_j$  distinguishes the fundamental bands, *i.e.* long-period ( $m = 0$ ), diurnal ( $m = 1$ ) and semidiurnal ( $m = 2$ ). The parameters  $H_j$  and  $\omega_j$  are used to obtain the most completely interpolated form

$$\Delta c = \sum_j a_{cj} \cos(\omega_j t + \chi_j - \phi_{cj}), \quad (2)$$

with

$$\begin{aligned} a_{cj} \cos \phi_{cj} &= H_j \left[ \frac{A_{ck} \cos \Phi_{ck}}{H_k} (1-p) + \frac{A_{c,k+1} \cos \Phi_{c,k+1}}{H_{k+1}} p \right], \\ a_{cj} \sin \phi_{cj} &= H_j \left[ \frac{A_{ck} \sin \Phi_{ck}}{H_k} (1-p) + \frac{A_{c,k+1} \sin \Phi_{c,k+1}}{H_{k+1}} p \right]. \end{aligned}$$

For each site, the amplitudes  $A_{ck}$  and phases  $\Phi_{ck}$ ,  $1 \leq k \leq 11$ , are taken from models such as those listed in Table 7.2. For clarity, symbols written with bars overhead designate tidal potential quantities associated with the small set of partial tides represented in the table. These are the semidiurnal waves  $M_2, S_2, N_2, K_2$ , the diurnal waves  $K_1, O_1, P_1, Q_1$ , and the long-period waves  $M_f, M_m$ , and  $S_{sa}$ .

Interpolation is possible only within a fundamental band, *i.e.* we demand

$$\bar{m}_k = m_j = \bar{m}_{k+1}. \quad (3)$$

Then

$$p = \frac{\omega_j - \bar{\omega}_k}{\bar{\omega}_{k+1} - \bar{\omega}_k}, \quad \bar{\omega}_k \leq \omega_j \leq \bar{\omega}_{k+1}.$$

If no  $\bar{\omega}_k$  or  $\bar{\omega}_{k+1}$  can be found meeting (3),  $p$  is set to zero or one, respectively.

A shorter form of (2) is obtained if the summation considers only the tidal species shown in Table 7.1 and corrections for the modulating effect of the lunar node. Then,

$$\Delta c = \sum_j f_j A_{cj} \cos(\omega_j t + \chi_j + u_j - \Phi_{cj}), \quad (4)$$

where  $f_j$  and  $u_j$  depend on the longitude of the lunar node. The astronomical arguments needed in (4) can be computed with subroutine ARG. The code for this subroutine can be obtained by anonymous ftp to <<sup>12</sup>>. The Tamura tide potential is available from the International Centre for Earth Tides, Observatoire Royal de Belgique, Bruxelles.

Information similar to that provided in Table 7.1 is available electronically from the ocean loading service site at <<sup>13</sup>>. Some precomputed tables are available at <<sup>14</sup>>.

Coefficients for stations that are farther away than 10 km from precomputed ones should always be recomputed.

The coefficients shown in Table 7.1 have been computed according to Scherneck (1991). Tangential displacements are to be taken positive in west and south directions. Tables are available derived from different ocean tide maps, GOT99.2 (Ray, 1999), CSR4.0 and CSR 3.0 (Eanes and Bettadpur, 1995), and models due to LeProvost *et al.* (1994). The automatic service computes coefficients selectably from a range of eleven ocean tide models, see Table 7.2.

The use of the most recent of these models is recommended (GOT00.2 for a TOPEX/POSEIDON derived solution, FES99 for a hydrodynamic solution). However, older models might be preferred for internal consistency. Since many space geodesy stations are inland or near coasts, the accuracy of the tide models in the shelf areas is more crucial than in the open sea. Refined coastlines have been derived from the topographic data sets ETOPO5 and Terrain Base (Row *et al.*, 1995) of the National Geophysical Data Center, Boulder, CO. Ocean tide mass budgets have been constrained using a uniform co-oscillating oceanic layer. Load convolution employed a disk-generating Green's function method (Farrell, 1972; Zschau, 1983; Scherneck, 1990). An assessment of the accuracy of the loading model is given in Scherneck (1993).

Additional contributions to ocean-induced displacement arise from the frequency dependence of the load Love numbers due to the Nearly Diurnal Free Wobble in the diurnal tidal band. The effect of this dependence may be taken into account, following Wahr and Sasao (1981), by incrementing the *body tide* Love numbers as explained further below.

<sup>12</sup>[maia.usno.navy.mil/conv2000/chapter7](http://maia.usno.navy.mil/conv2000/chapter7)

<sup>13</sup><http://www.oso.chalmers.se/~loading>

<sup>14</sup><http://www.oso.chalmers.se/~hgs/README.html>

Table 7.1 Sample of ocean loading table file. Each site record shows a header with the site name, the CDP monument number, geographic coordinates and comments. First three rows of numbers designate amplitudes (meter), radial, west, south, followed by three lines with the corresponding phase values (degrees).

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Columns designate partial tides  $M_2, S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_f, M_m,$  and  $S_{sa}$ .

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ONSALA60 7213

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\$\$ Computed by H.G. Scherneck, Uppsala University, 1989

\$\$ ONSALA 7213 lon/lat: 11.9263 57.3947

.00384	.00091	.00084	.00019	.00224	.00120	.00071	.00003	.00084	.00063	.00057
.00124	.00034	.00031	.00009	.00042	.00041	.00015	.00006	.00018	.00010	.00010
.00058	.00027	.00021	.00008	.00032	.00017	.00009	.00004	.00007	.00001	.00020
-56.0	-46.1	-90.7	-34.4	-44.5	-123.2	-49.6	178.4	14.9	37.3	24.6
75.4	97.6	40.8	94.8	119.0	25.4	98.7	-14.1	-177.0	-126.7	-175.8
84.2	131.3	77.7	103.9	17.2	-55.0	25.2	-165.0	173.3	121.8	91.3

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Table 7.2 Ocean tide models available at the automatic loading service.

Model code	Reference	Input	Resolution
SCHW	Schwiderski and Szeto (1981)	Tide gauge	$1^\circ \times 1^\circ$
CSR3.0, CSR4.0	Eanes (1994)	Topex/Poseidon Altim.	$1^\circ \times 1^\circ$
	Eanes and Bettadpur (1995)	T/P + LEPR loading	$0.5^\circ \times 0.5^\circ$
TPX0.5	Egbert <i>et al.</i> (1994)	inverse hydrodyn, solution from T/P Altim.	$256 \times 512$
FES94 (LEPR)	Le Provost <i>et al.</i> (1994)	numerical model	$0.5^\circ \times 0.5^\circ$
FES95			
FES98	Le Provost <i>et al.</i> (1998)	num. mdl. + assim. Altim.	$0.5^\circ \times 0.5^\circ$
FES99	Lefèvre <i>et al.</i> (2000)	numerical model	$0.25^\circ \times 0.25^\circ$
GOT99.2, GOT00.2	Ray (1999)	T/P	$0.5^\circ \times 0.5^\circ$
NAO99.b	Matsumoto <i>et al.</i> (2000)	num. + T/P assim.	$0.5^\circ \times 0.5^\circ$

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### 7.1.2 Effects of the Solid Earth Tides

Site displacements caused by tides of spherical harmonic degree and order ( $nm$ ) are characterized by the Love number  $h_{nm}$  and the Shida number  $l_{nm}$ . The effective values of these numbers depend on station latitude and tidal frequency (Wahr, 1981). The latitude dependence and a small interband variation are caused by the Earth's ellipticity and the Coriolis force due to Earth rotation. A strong frequency dependence within the diurnal band is produced by the Nearly Diurnal Free Wobble resonance associated with the free core nutation (FCN) in the wobbles of the Earth and its core regions which contribute to the tidal deformations via their centrifugal effects. Additionally, the resonance in the deformation due to ocean tidal loading, which is not included in the computations of the last section which use constant load Love numbers, may be represented in terms of effective contributions to  $h_{21}$  and  $l_{21}$ . A further frequency dependence, which is most pronounced in the long-period tidal band, arises from mantle anelasticity leading to corrections to the elastic Earth Love numbers. The contributions to the Love number parameters from anelasticity and ocean tidal loading as well as those from the centrifugal perturbations due to the wobbles have imaginary parts which cause the tidal displacements to lag slightly behind the tide generating potential. All these effects need to be taken into account when an accuracy of 1 mm is desired in determining station positions.

In order to account for the latitude dependence of the effective Love and Shida numbers, the representation in terms of multiple  $h$  and  $l$  parameters employed by Mathews *et al.* (1995) is used. In this representation, parameters  $h^{(0)}$  and  $l^{(0)}$  play the roles of  $h_{2m}$  and  $l_{2m}$ , while the latitude dependence is expressed in terms of additional parameters  $h^{(2)}$ ,  $h'$  and  $l^{(1)}$ ,  $l^{(2)}$ ,  $l'$ . These parameters are defined through their contributions to the site displacement as given by equations (5) below. Their numerical values as listed in the Conventions 1996 have since been revised, and the new values presented in Table 7.4 are used here. These values pertain to the elastic Earth and anelasticity models referred to in Chapter 6.

The vector displacement due to a tidal term of frequency  $f$  is given in terms of the several parameters by the following expressions that result from evaluation of the defining equation (6) of Mathews *et al.* (1995):

For a long-period tide of frequency  $f$ :

$$\Delta \vec{r}_f = \sqrt{\frac{5}{4\pi}} H_f \left\{ \begin{aligned} & \left[ h(\phi) \left( \frac{3}{2} \sin^2 \phi - \frac{1}{2} \right) + \sqrt{\frac{4\pi}{5}} h' \right] \cos \theta_f \hat{r} \\ & + 3l(\phi) \sin \phi \cos \phi \cos \theta_f \hat{n} \\ & + \cos \phi \left[ 3l^{(1)} \sin^2 \phi - \sqrt{\frac{4\pi}{5}} l' \right] \sin \theta_f \hat{e} \end{aligned} \right\}. \quad (5a)$$

For a diurnal tide of frequency  $f$ :

$$\Delta \vec{r}_f = -\sqrt{\frac{5}{24\pi}} H_f \left\{ \begin{aligned} & h(\phi) 3 \sin \phi \cos \phi \sin(\theta_f + \lambda) \hat{r} \\ & + \left[ 3l(\phi) \cos 2\phi - 3l^{(1)} \sin^2 \phi + \sqrt{\frac{24\pi}{5}} l' \right] \sin(\theta_f + \lambda) \hat{n} \\ & + \left[ \left( 3l(\phi) - \sqrt{\frac{24\pi}{5}} l' \right) \sin \phi - 3l^{(1)} \sin \phi \cos 2\phi \right] \cos(\theta_f + \lambda) \hat{e} \end{aligned} \right\}. \quad (5b)$$

For a semidiurnal tide of frequency  $f$ :

$$\Delta \vec{r}_f = \sqrt{\frac{5}{96\pi}} H_f \left\{ \begin{aligned} & [h(\phi) 3 \cos^2 \phi \cos(\theta_f + 2\lambda) \hat{r} \\ & - 6 \sin \phi \cos \phi [l(\phi) + l^{(1)}] \cos(\theta_f + 2\lambda) \hat{n} \\ & - 6 \cos \phi [l(\phi) + l^{(1)} \sin^2 \phi] \sin(\theta_f + 2\lambda) \hat{e} \end{aligned} \right\}. \quad (5c)$$

In the above expressions,

$$\begin{aligned} h(\phi) &= h^{(0)} + h^{(2)} [(3/2) \sin^2 \phi - 1/2], \\ l(\phi) &= l^{(0)} + l^{(2)} [(3/2) \sin^2 \phi - 1/2], \end{aligned} \quad (6)$$

- $H_f$  = amplitude (m) of the tidal term of frequency  $f$ ,
- $\phi$  = geocentric latitude of station,
- $\lambda$  = east longitude of station,
- $\theta_f$  = tide argument for tidal constituent with frequency  $f$ ,
- $\hat{r}$  = unit vector in the radial direction,
- $\hat{e}$  = unit vector in the east direction,
- $\hat{n}$  = unit vector at right angles to  $\hat{r}$  in the northward direction.

The convention used in defining the tidal amplitude  $H_f$  is as in Cartwright and Tayler (1971). To convert amplitudes defined according to other conventions that have been employed in recent more accurate tables, use the conversion factors given in Chapter 6, Table 6.5.

Equations (5) assume that the Love and Shida number parameters are all real. Generalization to the case of complex parameters is done simply by making the following replacements for the combinations  $L \cos(\theta_f + m\lambda)$  and  $L \sin(\theta_f + m\lambda)$ , wherever they occur in those equations:

$$L \cos(\theta_f + m\lambda) \rightarrow L^R \cos(\theta_f + m\lambda) - L^I \sin(\theta_f + m\lambda), \quad (7a)$$

$$L \sin(\theta_f + m\lambda) \rightarrow L^R \sin(\theta_f + m\lambda) + L^I \cos(\theta_f + m\lambda), \quad (7b)$$

where  $L$  is a generic symbol for  $h^{(0)}$ ,  $h^{(2)}$ ,  $h'$ ,  $l^{(0)}$ ,  $l^{(1)}$ ,  $l^{(2)}$ , and  $l'$ , and  $L^R$  and  $L^I$  stand for their respective real and imaginary parts.

The complex values of these 7 parameters are computed for the diurnal body tides from resonance formulae of the form given in equation (6) of Chapter 6 using the values listed in equation (7) of that chapter for the resonance frequencies  $\sigma_\alpha$  and those listed in Table 7.3 for the coefficients  $L_0$  and  $L_\alpha$  relating to each of the multiple  $h$  and  $l$  Love/Shida numbers. The manner in which  $\sigma_\alpha$  and the  $L_\alpha$  were computed is explained in Chapter 6, where mention is also made of the models used for the elastic Earth and for mantle anelasticity. As was noted in that chapter, the frequency dependence of the ocean tide contributions to certain Earth parameters in the equations of motion for the wobbles has the effect of making the resonance formulae inexact. The difference between the exact and resonance formula values is included in the tabulated values of  $h_{21}^{(0)}$  and  $l_{21}^{(0)}$  in Table 7.4. (The only case where this difference makes a contribution above the cut-off in Table 7.5a is in the radial displacement due to the  $\psi_1$  tide.) Also included in the values listed in Table 7.4 are the resonant ocean tidal loading corrections outlined in the next paragraph.

Table 7.3 Parameters in the Resonance Formulae for the Displacement Love Numbers.

$\alpha$	$h^{(0)}$		$h^{(2)}$	
	Re $L_\alpha$	Im $L_\alpha$	Re $L_\alpha$	Im $L_\alpha$
0	$0.60671 \times 10^{+0}$	$-0.2420 \times 10^{-2}$	$-0.615 \times 10^{-3}$	$-0.122 \times 10^{-4}$
1	$-0.15777 \times 10^{-2}$	$-0.7630 \times 10^{-4}$	$0.160 \times 10^{-5}$	$0.116 \times 10^{-6}$
2	$0.18053 \times 10^{-3}$	$-0.6292 \times 10^{-5}$	$0.201 \times 10^{-6}$	$0.279 \times 10^{-8}$
3	$-0.18616 \times 10^{-5}$	$0.1379 \times 10^{-6}$	$-0.329 \times 10^{-7}$	$-0.217 \times 10^{-8}$
$\alpha$	$l^{(0)}$		$l^{(1)}$	
	Re $L_\alpha$	Im $L_\alpha$	Re $L_\alpha$	Im $L_\alpha$
0	$.84963 \times 10^{-1}$	$-.7395 \times 10^{-3}$	$.121 \times 10^{-2}$	$.136 \times 10^{-6}$
1	$-.22107 \times 10^{-3}$	$-.9646 \times 10^{-5}$	$-.316 \times 10^{-5}$	$-.166 \times 10^{-6}$
2	$-.54710 \times 10^{-5}$	$-.2990 \times 10^{-6}$	$.272 \times 10^{-6}$	$-.858 \times 10^{-8}$
3	$-.29904 \times 10^{-7}$	$-.7717 \times 10^{-8}$	$-.545 \times 10^{-8}$	$.827 \times 10^{-11}$
$\alpha$	$l^{(2)}$		$l'$	
	Re $L_\alpha$	Im $L_\alpha$	Re $L_\alpha$	Im $L_\alpha$
0	$.19334 \times 10^{-3}$	$-.3819 \times 10^{-5}$	$-.221 \times 10^{-3}$	$-.474 \times 10^{-7}$
1	$-.50331 \times 10^{-6}$	$-.1639 \times 10^{-7}$	$.576 \times 10^{-6}$	$.303 \times 10^{-7}$
2	$-.66460 \times 10^{-8}$	$.5076 \times 10^{-9}$	$.128 \times 10^{-6}$	$-.378 \times 10^{-8}$
3	$.10372 \times 10^{-7}$	$.7511 \times 10^{-9}$	$-.655 \times 10^{-8}$	$-.291 \times 10^{-9}$

Site displacements caused by solid Earth deformations due to ocean tidal loading have been dealt with in the first section of this chapter. Constant nominal values were assumed for the load Love numbers in computing these. The values used for tides of degree 2 were

$$h_2^{(nom)} = -1.001, \quad l_2^{(nom)} = 0.0295, \quad k_2^{(nom)} = -0.3075.$$

Since resonances in the diurnal band cause the values of the load Love numbers too to vary, corrections need to be applied to the results of the first section. These corrections can be expressed in terms of effective ocean tide contributions  $\delta h^{(OT)}$  and  $\delta l^{(OT)}$  to the respective body tide Love numbers  $h_{21}^{(0)}$  and  $l_{21}^{(0)}$ .  $\delta h^{(OT)}$  and  $\delta l^{(OT)}$  are given by expressions of the form (8) of Chapter 6, with appropriate replacements. They were computed using the same ocean tide admittances as in that chapter, and using the resonance parameters listed in Table 6.2 for the load Love numbers; they are included in the values listed in Table 7.4 under  $h^{(0)R}$  and  $h^{(0)I}$  for the diurnal tides.

The variation of  $h_{20}^{(0)}$  and  $l_{20}^{(0)}$  across the zonal tidal band,  $(nm) = (20)$ , due to mantle anelasticity, is described by the formulae

$$h_{20}^{(0)} = 0.5998 - 9.96 \times 10^{-4} \left\{ \cot \frac{\alpha\pi}{2} \left[ 1 - \left( \frac{f_m}{f} \right)^\alpha \right] + i \left( \frac{f_m}{f} \right)^\alpha \right\}, \quad (8a)$$

$$l_{20}^{(0)} = 0.0831 - 3.01 \times 10^{-4} \left\{ \cot \frac{\alpha\pi}{2} \left[ 1 - \left( \frac{f_m}{f} \right)^\alpha \right] + i \left( \frac{f_m}{f} \right)^\alpha \right\} \quad (8b)$$

on the basis of the anelasticity model already referred to. Here  $f$  is the frequency of the zonal tidal constituent,  $f_m$  is the reference frequency equivalent to a period of 200 s, and  $\alpha = 0.15$ .

Table 7.4 lists the values of  $h^{(0)}$ ,  $h^{(2)}$ ,  $h'$ ,  $l^{(0)}$ ,  $l^{(1)}$ ,  $l^{(2)}$ , and  $l'$  for those tidal frequencies for which they are needed for use in the computational procedure described below. The tidal frequencies shown in the table are in cycles per sidereal day (*cpsd*). Periods, in solar days, of the nutations associated with the diurnal tides are also shown.

Computation of the variations of station coordinates due to solid Earth tides, like that of geopotential variations, is done most efficiently by the use of a two-step procedure. The evaluations in the first step use the expression in the time domain for the full degree 2 tidal potential or for the parts that pertain to particular bands ( $m = 0, 1$ , or  $2$ ). Nominal values common to all the tidal constituents involved in the potential and to all stations are used for the Love and Shida numbers  $h_{2m}$  and  $l_{2m}$  in this step. They are chosen with reference to the values in Table 7.4 so as to minimize the computational effort needed in Step 2. Along with expressions for the dominant contributions from  $h^{(0)}$  and  $l^{(0)}$  to the tidal displacements, relatively small contributions from some of the other parameters are included in Step 1 for reasons of computational efficiency. The displacements caused by the degree 3 tides are also computed in the first step, using constant values for  $h_3$  and  $l_3$ .

Corrections to the results of the first step are needed to take account of the frequency dependent deviations of the Love and Shida numbers from their respective nominal values, and also to compute the out-of-phase contributions from the zonal tides. Computations of these corrections constitute Step 2. The total displacement due to the tidal potential is the sum of the displacements computed in Steps 1 and 2.

The full scheme of computation is outlined in the chart on page 79.

Table 7.4 Displacement Love number parameters for degree 2 tides. Super-  
scripts  $R$  and  $I$  identify the real and imaginary parts, respectively.

Name	Period	Frequency	$h^{(0)R}$	$h^{(0)I}$	$h^{(2)}$	$h'$	
Semidiurnal		-2 <i>cpsd</i>	.6078	-.0022	-.0006		
Diurnal							
2Q <sub>1</sub>	6.86	0.85461	.6039	-.0027	-.0006		
$\sigma_1$	7.10	0.85946	.6039	-.0026	-.0006		
135,645	9.12	0.89066	.6036	-.0026	-.0006		
Q <sub>1</sub>	9.13	0.89080	.6036	-.0026	-.0006		
$\rho_1$	9.56	0.89565	.6035	-.0026	-.0006		
145,545	13.63	0.92685	.6028	-.0025	-.0006		
O <sub>1</sub>	13.66	0.92700	.6028	-.0025	-.0006		
$\tau_1$	14.77	0.93246	.6026	-.0025	-.0006		
N $\tau_1$	23.94	0.95835	.6011	-.0024	-.0006		
NO <sub>1</sub>	27.55	0.96381	.6005	-.0023	-.0006		
$\chi_1$	31.81	0.96865	.5998	-.0023	-.0006		
$\pi_1$	121.75	0.99181	.5878	-.0015	-.0006		
P <sub>1</sub>	182.62	0.99454	.5817	-.0011	-.0006		
S <sub>1</sub>	365.26	0.99727	.5692	-.0004	-.0006		
165,545	6798.38	0.99985	.5283	.0023	-.0007		
K <sub>1</sub>	infinity	1.00000	.5236	.0030	-.0007		
165,565	-6798.38	1.00015	.5182	.0036	-.0007		
165,575	-3399.19	1.00029	.5120	.0043	-.0007		
$\psi_1$	-365.26	1.00273	1.0569	.0036	-.0001		
166,564	-346.64	1.00288	.9387	-.0050	-.0003		
$\phi_1$	-182.62	1.00546	.6645	-.0059	-.0006		
$\theta_1$	-31.81	1.03135	.6117	-.0030	-.0006		
J <sub>1</sub>	-27.55	1.03619	.6108	-.0030	-.0006		
OO <sub>1</sub>	-13.66	1.07300	.6080	-.0028	-.0006		
Long period							
55,565	6798.38	.000147	.6344	-.0093	-.0006	.0001	
$S_{sa}$	182.62	.005461	.6182	-.0054	-.0006	.0001	
$M_m$	27.55	.036193	.6126	-.0041	-.0006	.0001	
$M_f$	13.66	.073002	.6109	-.0037	-.0006	.0001	
75,565	13.63	.073149	.6109	-.0037	-.0006	.0001	
Name	Period	Frequency	$l^{(0)R}$	$l^{(0)I}$	$l^{(1)}$	$l^{(2)}$	$l'$
Semidiurnal		-2 <i>cpsd</i>	.0847	-.0007	.0024	.0002	
Diurnal							
Q <sub>1</sub>	9.13	0.89080	.0846	-.0006	.0012	.0002	-.0002
145,545	13.63	0.92685	.0846	-.0006	.0012	.0002	-.0002
O <sub>1</sub>	13.66	0.92700	.0846	-.0006	.0012	.0002	-.0002
NO <sub>1</sub>	27.55	0.96381	.0847	-.0006	.0012	.0002	-.0002
P <sub>1</sub>	182.62	0.99454	.0853	-.0006	.0012	.0002	-.0002
165,545	6798.38	0.99985	.0869	-.0006	.0011	.0002	-.0003
K <sub>1</sub>	infinity	1.00000	.0870	-.0006	.0011	.0002	-.0003
165,565	-6798.38	1.00015	.0872	-.0006	.0011	.0002	-.0003
$\psi_1$	-365.26	1.00273	.0710	-.0020	.0019	.0002	.0001
$\phi_1$	-182.62	1.00546	.0828	-.0007	.0013	.0002	-.0002
J <sub>1</sub>	-27.55	1.03619	.0845	-.0006	.0012	.0002	-.0002
OO <sub>1</sub>	-13.66	1.07300	.0846	-.0006	.0012	.0002	-.0002
Long period							
55,565	6798.38	.000147	.0936	-.0028	.0000	.0002	
$S_{sa}$	182.62	.005461	.0886	-.0016	.0000	.0002	
$M_m$	27.55	.036193	.0870	-.0012	.0000	.0002	
$M_f$	13.66	.073002	.0864	-.0011	.0000	.0002	
75,565	13.63	.073149	.0864	-.0011	.0000	.0002	

## CORRECTIONS FOR THE STATION TIDAL DISPLACEMENTS

**Step 1:** Corrections to be computed in the time domain

in-phase	for degree 2 and 3	Nominal values
	. for degree 2 → eq (9)	$h_2 \rightarrow h(\phi) = h^{(0)} + h^{(2)}[(3 \sin^2 \phi - 1)/2]$ $l_2 \rightarrow l(\phi) = l^{(0)} + l^{(2)}[(3 \sin^2 \phi - 1)/2]$ $h^{(0)} = 0.6078, h^{(2)} = -0.0006; l^{(0)} = 0.0847, l^{(2)} = 0.0002$
	. for degree 3 → eq (10)	$h_3 = 0.292$ and $l_3 = 0.015$
out-of-phase	for degree 2 only	Nominal values
	. diurnal tides → eq (14)	$h^I = -0.0025$ and $l^I = -0.0007$
	. semidiurnal tides → eq (15)	$h^I = -0.0022$ and $l^I = -0.0007$
contribution	from latitude dependence	Nominal values
	. diurnal tides → eq (12)	$l^{(1)} = 0.0012$
	. semidiurnal tides → eq (13)	$l^{(1)} = 0.0024$

**Step 2:** Corrections to be computed in the frequency domain and to be added to results of Step 1

in-phase	for degree 2	
	. diurnal tides → eqs (16)	→ Sum over all the components of Table 7.5a
	. semidiurnal tides	negligible
in-phase	and out-of-phase for degree 2	
	. long-period tides → eqs (17)	→ Sum over all the components of Table 7.5b

**Displacement due to degree 2 tides, with nominal values for  $h_{2m}^{(0)}$  and  $l_{2m}^{(0)}$** 

The first stage of the Step 1 calculations employs real nominal values  $h_2$  and  $l_2$  common to all the degree 2 tides for the Love and Shida numbers. It is found to be computationally most economical to choose these to be the values for the semidiurnal tides (which have very little intra-band variation). On using the nominal values, the vector displacement of the station due to the degree 2 tides is given by

$$\Delta \vec{r} = \sum_{j=2}^3 \frac{GM_j R_e^4}{GM_{\oplus} R_j^3} \left\{ h_2 \hat{r} \left( \frac{3}{2} (\hat{R}_j \cdot \hat{r})^2 - \frac{1}{2} \right) + 3l_2 (\hat{R}_j \cdot \hat{r}) [\hat{R}_j - (\hat{R}_j \cdot \hat{r}) \hat{r}] \right\}, \quad (9)$$

where  $h_{22}^{(0)}$  and  $l_{22}^{(0)}$  of the semidiurnal tides are chosen as the nominal values  $h_2$  and  $l_2$ . The out-of-phase displacements due to the imaginary parts of the Love numbers are dealt with separately below. In equation (9),

$GM_j$	=	gravitational parameter for the Moon ( $j = 2$ ) or the Sun ( $j = 3$ ),
$GM_{\oplus}$	=	gravitational parameter for the Earth,
$\hat{R}_j, R_j$	=	unit vector from the geocenter to Moon or Sun and the magnitude of that vector,
$R_e$	=	Earth's equatorial radius,
$\hat{r}, r$	=	unit vector from the geocenter to the station and the magnitude of that vector,
$h_2$	=	nominal degree 2 Love number,
$l_2$	=	nominal degree 2 Shida number.

Note that the part proportional to  $h_2$  gives the radial (not vertical) component of the tide-induced station displacement, and the terms in  $l_2$  represent the vector displacement transverse to the radial direction (and not in the horizontal plane).

The computation just described may be generalized to include the latitude dependence arising through  $h^{(2)}$  by simply adding  $h^{(2)}[(3/2)\sin^2\phi - (1/2)]$  to the constant nominal value given above, with  $h^{(2)} = -0.0006$ . The addition of a similar term (with  $l^{(2)} = 0.0002$ ) to the nominal value of  $l_2$  takes care of the corresponding contribution to the transverse displacement. The resulting incremental displacements are small, not exceeding 0.4 mm radially and 0.2 mm in the transverse direction.

### Displacements due to degree 3 tides

The Love numbers of the degree 3 tides may be taken as real and constant in computations to the degree of accuracy aimed at here. The vector displacement due to these tides is then given by

$$\Delta\vec{r} = \sum_{j=2}^3 \frac{GM_j R_e^5}{GM_{\oplus} R_j^4} \left\{ h_3 \hat{r} \left( \frac{5}{2} (\hat{R}_j \cdot \hat{r})^3 - \frac{3}{2} (\hat{R}_j \cdot \hat{r}) \right) + l_3 \left( \frac{15}{2} (\hat{R}_j \cdot \hat{r})^2 - \frac{3}{2} \right) [\hat{R}_j - (\hat{R}_j \cdot \hat{r})\hat{r}] \right\}. \quad (10)$$

Only the Moon's contribution ( $j = 2$ ) need be computed, the term due to the Sun being quite ignorable. The transverse part of the displacement (10) does not exceed 0.2 mm, but the radial displacement can reach 1.7 mm.

### Contributions to the transverse displacement due to the $l^{(1)}$ term

The imaginary part of  $l^{(1)}$  is completely ignorable, as is the intra-band variation of  $\text{Re } l^{(1)}$ ; and  $l^{(1)}$  is effectively zero in the zonal band.

In the expressions given below, and elsewhere in this chapter,

$$\begin{aligned} \Phi_j &= \text{body fixed geocentric latitude of Moon or Sun, and} \\ \lambda_j &= \text{body fixed east longitude (from Greenwich) of Moon or Sun.} \end{aligned}$$

The following formulae may be employed when the use of Cartesian coordinates  $X_j, Y_j, Z_j$  of the body relative to the terrestrial reference frame is preferred:

$$P_2^0(\sin \Phi_j) = \frac{1}{R_j^2} \left( \frac{3}{2} Z_j^2 - \frac{1}{2} R_j^2 \right), \quad (11a)$$

$$P_2^1(\sin \Phi_j) \cos \lambda_j = \frac{3X_j Z_j}{R_j^2}, \quad (11b)$$

$$P_2^1(\sin \Phi_j) \sin \lambda_j = \frac{3Y_j Z_j}{R_j^2},$$

$$P_2^2(\sin \Phi_j) \cos 2\lambda_j = \frac{3}{R_j^2} (X_j^2 - Y_j^2), \quad (11c)$$

$$P_2^2(\sin \Phi_j) \sin 2\lambda_j = \frac{6}{R_j^2} X_j Y_j.$$

Contribution from the diurnal band (with  $l^{(1)} = 0.0012$ ):

$$\delta\vec{t} = -l^{(1)} \sin \phi \sum_{j=2}^3 \frac{GM_j R_e^4}{GM_{\oplus} R_j^3} P_2^1(\sin \Phi_j) [\sin \phi \cos(\lambda - \lambda_j) \hat{n} - \cos 2\phi \sin(\lambda - \lambda_j) \hat{e}]. \quad (12)$$

Contribution from the semidiurnal band (with  $l^{(1)} = 0.0024$ ):

$$\delta\vec{t} = -\frac{1}{2} l^{(1)} \sin \phi \cos \phi \sum_{j=2}^3 \frac{GM_j R_e^4}{GM_{\oplus} R_j^3} P_2^2(\sin \Phi_j) [\cos 2(\lambda - \lambda_j) \hat{n} + \sin \phi \sin 2(\lambda - \lambda_j) \hat{e}]. \quad (13)$$

The contributions of the  $l^{(1)}$  term to the transverse displacements caused by the diurnal and semidiurnal tides could be up to 0.8 mm and 1.0 mm respectively.

**Out of phase contributions from the imaginary parts of  $h_{2m}^{(0)}$  and  $l_{2m}^{(0)}$**

In the following,  $h^I$  and  $l^I$  stand for the imaginary parts of  $h_{2m}^{(0)}$  and  $l_{2m}^{(0)}$ . Contributions  $\delta r$  to radial and  $\delta \vec{t}$  to transverse displacements from diurnal tides (with  $h^I = -0.0025$ ,  $l^I = -0.0007$ ):

$$\delta r = -\frac{3}{4}h^I \sum_{j=2}^3 \frac{GM_j R_e^4}{GM_{\oplus} R_j^3} \sin 2\Phi_j \sin 2\phi \sin(\lambda - \lambda_j), \quad (14a)$$

$$\delta \vec{t} = -\frac{3}{2}l^I \sum_{j=2}^3 \frac{GM_j R_e^4}{GM_{\oplus} R_j^3} \sin 2\Phi_j [\cos 2\phi \sin(\lambda - \lambda_j) \hat{n} + \sin \phi \cos(\lambda - \lambda_j) \hat{e}]. \quad (14b)$$

Contributions from semidiurnal tides (with  $h^I = -0.0022$ ,  $l^I = -0.0007$ ):

$$\delta r = -\frac{3}{4}h^I \sum_{j=2}^3 \frac{GM_j R_e^4}{GM_{\oplus} R_j^3} \cos^2 \Phi_j \cos^2 \phi \sin 2(\lambda - \lambda_j), \quad (15a)$$

$$\delta \vec{t} = \frac{3}{4}l^I \sum_{j=2}^3 \frac{GM_j R_e^4}{GM_{\oplus} R_j^3} \cos^2 \Phi_j [\sin 2\phi \sin 2(\lambda - \lambda_j) \hat{n} - 2 \cos \phi \cos 2(\lambda - \lambda_j) \hat{e}]. \quad (15b)$$

The out-of-phase contributions from the zonal tides has no closed expression in the time domain.

Computations of Step 2 are to take account of the intra-band variation of  $h_{2m}^{(0)}$  and  $l_{2m}^{(0)}$ . Variations of the imaginary parts are negligible except as stated below. For the zonal tides, however, the contributions from the imaginary part have to be computed in Step 2.

### Correction for frequency dependence of the Love and Shida numbers

(a) Contributions from the diurnal band

Corrections to the radial and transverse station displacements  $\delta r$  and  $\delta \vec{t}$  due to a diurnal tidal term of frequency  $f$  are obtainable from equation (5b):

$$\delta r = [\delta R_f^{(ip)} \sin(\theta_f + \lambda) + \delta R_f^{(op)} \cos(\theta_f + \lambda)] \sin 2\phi, \quad (16a)$$

$$\delta \vec{t} = [\delta T_f^{(ip)} \cos(\theta_f + \lambda) - \delta T_f^{(op)} \sin(\theta_f + \lambda)] \sin \phi \hat{e} + [\delta T_f^{(ip)} \sin(\theta_f + \lambda) + \delta T_f^{(op)} \cos(\theta_f + \lambda)] \cos 2\phi \hat{n}, \quad (16b)$$

where

$$\begin{pmatrix} \delta R_f^{(ip)} \\ \delta R_f^{(op)} \end{pmatrix} = -\frac{3}{2} \sqrt{\frac{5}{24\pi}} H_f \begin{pmatrix} \delta h_f^R \\ \delta h_f^I \end{pmatrix}, \quad (16c)$$

$$\begin{pmatrix} \delta T_f^{(ip)} \\ \delta T_f^{(op)} \end{pmatrix} = -3 \sqrt{\frac{5}{24\pi}} H_f \begin{pmatrix} \delta l_f^R \\ \delta l_f^I \end{pmatrix},$$

and

$\delta h_f^R$  and  $\delta h_f^I$  are the differences of  $h^{(0)R}$  and  $h^{(0)I}$  at frequency  $f$  from the nominal values  $h_2$  and  $h^I$  used in equations (9) and (14a), respectively,

$\delta l_f^R$  and  $\delta l_f^I$  are the differences of  $l^{(0)R}$  and  $l^{(0)I}$  at frequency  $f$  from the nominal values  $l_2$  and  $l^I$  used in equations (9) and (14b), respectively.

Table 7.5a Corrections due to the frequency variation of Love and Shida numbers for diurnal tides. Units: mm. All terms with radial correction  $\geq 0.05$  mm are shown. Nominal values are  $h_2 = 0.6078$  and  $l_2 = 0.0847$  for the real parts, and  $h^I = -0.0025$  and  $l^I = -0.0007$  for the imaginary parts. Frequencies shown are in degrees per hour.

Name	Frequency	Doodson	$\tau$	$s$	$h$	$p$	$N'$	$p_s$	$\ell$	$\ell'$	$F$	$D$	$\Omega$	$\Delta R_f^{(ip)}$	$\Delta R_f^{(op)}$	$\Delta T_f^{(ip)}$	$\Delta T_f^{(op)}$
Q <sub>1</sub>	13.39866	135,655	1	-2	0	1	0	0	1	0	2	0	2	-0.08	0.00	-0.01	0.01
	13.94083	145,545	1	-1	0	0	-1	0	0	0	2	0	1	-0.10	0.00	0.00	0.00
O <sub>1</sub>	13.94303	145,555	1	-1	0	0	0	0	0	0	2	0	2	-0.51	0.00	-0.02	0.03
NO <sub>1</sub>	14.49669	155,655	1	0	0	1	0	0	1	0	0	0	0	0.06	0.00	0.00	0.00
$\pi_1$	14.91787	162,556	1	1	-3	0	0	1	0	1	2	-2	2	-0.06	0.00	0.00	0.00
P <sub>1</sub>	14.95893	163,555	1	1	-2	0	0	0	0	0	2	-2	2	-1.23	-0.07	0.06	0.01
	15.03886	165,545	1	1	0	0	-1	0	0	0	0	0	-1	-0.22	0.01	0.01	0.00
K <sub>1</sub>	15.04107	165,555	1	1	0	0	0	0	0	0	0	0	0	12.00	-0.78	-0.67	-0.03
	15.04328	165,565	1	1	0	0	1	0	0	0	0	0	1	1.73	-0.12	-0.10	0.00
$\psi_1$	15.08214	166,554	1	1	1	0	0	-1	0	-1	0	0	0	-0.50	-0.01	0.03	0.00
$\phi_1$	15.12321	167,555	1	1	2	0	0	0	0	0	-2	2	-2	-0.11	0.01	0.01	0.00

## (b) Contributions from the long-period band

Corrections  $\delta r$  and  $\delta \vec{t}$  due to a zonal tidal term of frequency  $f$  include both in-phase ( $ip$ ) and out-of-phase ( $op$ ) parts. From equations (5a) and (7) one finds that

$$\delta r = \left( \frac{3}{2} \sin^2 \phi - \frac{1}{2} \right) (\delta R_f^{(ip)} \cos \theta_f + \delta R_f^{(op)} \sin \theta_f), \quad (17a)$$

and

$$\delta \vec{t} = (\delta T_f^{(ip)} \cos \theta_f + \delta T_f^{(op)} \sin \theta_f) \sin 2\phi \hat{n}, \quad (17b)$$

where

$$\begin{pmatrix} \delta R_f^{(ip)} \\ \delta R_f^{(op)} \end{pmatrix} = \sqrt{\frac{5}{4\pi}} H_f \begin{pmatrix} \delta h_f^R \\ -\delta h_f^I \end{pmatrix}, \quad (17c)$$

and

$$\begin{pmatrix} \delta T_f^{(ip)} \\ \delta T_f^{(op)} \end{pmatrix} = \frac{3}{2} \sqrt{\frac{5}{4\pi}} H_f \begin{pmatrix} \delta l_f^R \\ -\delta l_f^I \end{pmatrix}.$$

Table 7.5b Corrections due to frequency variation of Love and Shida numbers for zonal tides. Units: mm. All terms with radial correction  $\geq 0.05$  mm are shown. Nominal values are  $h = 0.6078$  and  $l = 0.0847$ .

Name	Frequency	Doodson	$\tau$	$s$	$h$	$p$	$N'$	$p_s$	$\ell$	$\ell'$	$F$	$D$	$\Omega$	$\Delta R_f^{(ip)}$	$\Delta R_f^{(op)}$	$\Delta T_f^{(ip)}$	$\Delta T_f^{(op)}$
$S_{sa}$	0.00221	55,565	0	0	0	0	1	0	0	0	0	0	1	0.47	0.16	0.23	0.07
	0.08214	57,555	0	0	2	0	0	0	0	0	-2	2	-2	-0.20	-0.11	-0.12	-0.05
$M_m$	0.54438	65,455	0	1	0	-1	0	0	-1	0	0	0	0	-0.11	-0.09	-0.08	-0.04
$M_f$	1.09804	75,555	0	2	0	0	0	0	0	0	-2	0	-2	-0.13	-0.15	-0.11	-0.07
	1.10024	75,565	0	2	0	0	1	0	0	0	-2	0	-1	-0.05	-0.06	-0.05	-0.03

Values of  $\Delta R_f$  and  $\Delta T_f$  listed in Tables 7.5a and 7.5b are for the constituents that must be taken into account to ensure an accuracy of 1 mm.

A FORTRAN program for computing the various corrections is available at <<sup>15</sup>>.

<sup>15</sup>ftp://omaftp.oma.be/dist/astro/dehant/IERS/

### 7.1.3 Permanent deformation

The tidal model described above does contain in principle a time independent part so that the coordinates obtained by taking into account this model in the analysis will be “conventional tide free” values. (Note that they do *not* correspond to what would be observed in the absence of tidal perturbation. See the discussion in Chapter 1.) This section allows a user to compute “mean tide” coordinates from “conventional tide free” coordinates.

Specifically, the degree 2 zonal tide generating potential includes a spectral component of zero frequency and amplitude  $H_0 = -0.31460$  m, and its effect enters the tidal displacement model through the time independent component of the expression (9). Evaluation of this component may be done using equations (5a) and (6) with  $H_f = H_0, \theta_f = 0$ , and with the same nominal values for the Love number parameters as were used in Step 1:  $h_2 = 0.6078, l_2 = 0.0847$  along with  $h^{(2)} = -0.0006$  and  $l^{(2)} = 0.0002$ . One finds the radial component of the permanent displacement according to (9) to be

$$[-0.1206 + 0.0001P_2(\sin \phi)]P_2(\sin \phi) \text{ m}, \quad (18a)$$

and the transverse component to be

$$[-0.0252 - 0.0001P_2(\sin \phi)] \sin 2\phi \text{ m} \quad (18b)$$

northwards, where  $P_2(\sin \phi) = (3 \sin^2 \phi - 1)/2$ .

These are the components of the vector to be added to the “conventional tide free” computed tide-corrected position to obtain the “mean tide” position. The radial component of this restitution to obtain the “mean tide” values amounts to about  $-12$  cm at the poles and about  $+6$  cm at the equator.

### 7.1.4 Rotational Deformation due to Polar Motion

The variation of station coordinates caused by the pole tide can amount to a couple of centimeters and needs to be taken into account.

Let us choose  $\hat{x}, \hat{y}$  and  $\hat{z}$  as a terrestrial system of reference. The  $\hat{z}$  axis is oriented along the Earth’s mean rotation axis, the  $\hat{x}$  axis is in the direction of the adopted origin of longitude and the  $\hat{y}$  axis is orthogonal to the  $\hat{x}$  and  $\hat{z}$  axes and in the plane of the  $90^\circ$  E meridian.

The centrifugal potential caused by the Earth’s rotation is

$$V = \frac{1}{2}[r^2|\vec{\Omega}|^2 - (\vec{r} \cdot \vec{\Omega})^2], \quad (19)$$

where  $\vec{\Omega} = \Omega(m_1 \hat{x} + m_2 \hat{y} + (1 + m_3) \hat{z})$ .  $\Omega$  is the mean angular velocity of rotation of the Earth;  $m_1, m_2$  describe the time dependent offset of the instantaneous rotation pole from the mean, and  $m_3$ , the fractional variation in the rotation rate;  $r$  is the geocentric distance to the station.

Neglecting the variations in  $m_3$  which induce displacements that are below the mm level, the  $m_1$  and  $m_2$  terms give a first order perturbation in the potential  $V$  (Wahr, 1985)

$$\Delta V(r, \theta, \lambda) = -\frac{\Omega^2 r^2}{2} \sin 2\theta (m_1 \cos \lambda + m_2 \sin \lambda). \quad (20)$$

The radial displacement  $S_r$  and the horizontal displacements  $S_\theta$  and  $S_\lambda$  (positive upwards, south and east respectively in a horizon system at the station) due to  $\Delta V$  are obtained using the formulation of tidal Love numbers (Munk and MacDonald, 1960):

$$S_r = h_2 \frac{\Delta V}{g}, \quad S_\theta = \frac{\ell_2}{g} \partial_\theta \Delta V, \quad S_\lambda = \frac{\ell_2}{g} \frac{1}{\sin \theta} \partial_\lambda \Delta V. \quad (21)$$

The position of the Earth's mean rotation pole has a secular variation, and its coordinates in the Terrestrial Reference Frame discussed in Chapter 4 are given, in terms of the polar motion variables  $(x_p, y_p)$  defined in Chapter 5, by appropriate running averages  $\bar{x}_p$  and  $-\bar{y}_p$ . Then

$$m_1 = x_p - \bar{x}_p, \quad m_2 = -(y_p - \bar{y}_p). \quad (22)$$

For the most accurate results, estimates of the mean pole should be used. These are provided by the IERS Earth Orientation Centre and are made available at <<sup>16</sup>>. It is also possible to approximate the pole path by a linear trend. The estimates below are derived from the same IERS Earth Orientation Centre data.

$$\bar{x}_p(t) = \bar{x}_p(t_0) + (t - t_0)\dot{\bar{x}}_p(t_0), \quad \bar{y}_p(t) = \bar{y}_p(t_0) + (t - t_0)\dot{\bar{y}}_p(t_0), \quad (23a)$$

$$\bar{x}_p(t_0) = 0.054, \quad \dot{\bar{x}}_p(t_0) = 0.00083, \quad \bar{y}_p(t_0) = 0.357, \quad \dot{\bar{y}}_p(t_0) = 0.00395, \quad (23b)$$

where  $\bar{x}_p, \bar{y}_p$  are in arcseconds, their rates are in arcseconds per year, and  $t_0$  is 2000.

Using Love number values appropriate to the frequency of the pole tide ( $h = 0.6027$ ,  $l = 0.0836$ ) and  $r = a = 6.378 \times 10^6$  m, one finds

$$\begin{aligned} S_r &= -32 \sin 2\theta (m_1 \cos \lambda + m_2 \sin \lambda) \text{ mm}, \\ S_\theta &= -9 \cos 2\theta (m_1 \cos \lambda + m_2 \sin \lambda) \text{ (mm)}, \\ S_\lambda &= 9 \cos \theta (m_1 \sin \lambda - m_2 \cos \lambda) \text{ mm}, \end{aligned} \quad (24)$$

with  $m_1$  and  $m_2$  given in seconds of arc.

Taking into account that  $m_1$  and  $m_2$  vary, at most, 0.8 arcsec, the maximum radial displacement is approximately 25 mm, and the maximum horizontal displacement is about 7 mm.

If  $X$ ,  $Y$ , and  $Z$  are Cartesian coordinates of a station in a right-handed equatorial coordinate system, the changes in them due to polar motion are

$$[dX, dY, dZ]^T = R^T [S_\theta, S_\lambda, S_r]^T, \quad (25)$$

where

$$R = \begin{pmatrix} \cos \theta \cos \lambda & \cos \theta \sin \lambda & -\sin \theta \\ -\sin \lambda & \cos \lambda & 0 \\ \sin \theta \cos \lambda & \sin \theta \sin \lambda & \cos \theta \end{pmatrix}. \quad (26)$$

### 7.1.5 Atmospheric Loading

Temporal variations in the geographic distribution of atmospheric mass load the Earth and deform its surface. For example, pressure variations on the order of 20 HPa (and even larger) at mid-latitudes, are observed in synoptic pressure systems with length scales for 1000-2000 km and periods of approximately two weeks. Seasonal pressure changes due to air mass movements between the continents and oceans can have amplitudes of up to 10 HPa in particular over the large land masses of the Northern Hemisphere. At the interannual periods, basin-wide air pressure signals with amplitudes of several HPa also contribute to the spectrum of the loading signal.

Other surface loads due to changes in snow and ice cover, soil moisture and groundwater, as well as ocean-bottom pressure also contribute to surface displacements. For example, at seasonal time scales, it is expected that the contribution of hydrological loads to surface displacements exceeds the one from air pressure (Blewitt *et al.*, 2001). However, while the atmospheric load is fairly well known from global air pressure data sets, no sufficient models for ocean bottom pressure, snow and soil moisture exists at this time. Therefore, in the following, focus is on atmospheric loading. However, the discussion applies also to any other surface load.

<sup>16</sup>ftp://maia.usno.navy.mil/conv2000/chapter7/annual.pole

Theoretical studies by Rabbel and Zschau (1985), Rabbel and Schuh (1986), vanDam and Wahr (1987), and Manabe *et al.* (1991) demonstrate that vertical crustal displacements of up to 25 mm are possible at mid-latitude stations due to synoptic pressure systems. Annual signals in the vertical are on the order of 1-2 mm but maximum signals of more than 3 mm are possible over large parts of Asia, Antarctica, Australia and Greenland (Mangarotti *et al.*, 2001; Dong *et al.*, 2002). Pressure loading effects are larger at higher latitude sites due to the more intensive weather systems (larger in amplitude and more spatially coherent) found there. Effects are smaller at mid-latitude sites and at locations within 500 km of the sea or ocean due to the inverted barometer response of the ocean. In all cases, horizontal crustal deformations are about one-third the amplitude of the vertical effects.

Two basic methods for computing atmospheric loading corrections to geodetic data have been applied so far: 1) using geophysical models or simple approximations derived from these models and 2) using empirical models based on site-dependent data.

The standard geophysical model approach is based on the estimation of atmospheric loading effects (vertical and horizontal deformations, gravity, tilt and strain) via the convolution of Green's functions with a global surface pressure field. The geophysical approach is analogous to methods used to calculate ocean tidal loading effects. However, due to the continuous spectrum of the atmospheric pressure variations, the computation of the atmospheric loading signal must be carried out in the time domain. The major advantage of the geophysical model approach is that loading effects can be computed in a standardized way for any point on the Earth's surface more or less instantaneously. The geophysical approach currently suffers from a number of problems including: the requirement of a global pressure data set, a minimum of 24 hours in time delay in the availability of the global pressure data set, limitations of the pressure data itself (low temporal and spatial resolution), uncertainties in the Green's functions and uncertainties in the ocean response model.

In the empirical approach, site-dependent pressure loading effects are computed by determining the fit of local pressure variations to the geodetic observations of the vertical crustal motion. This approach is likely to produce better results (than the geophysical approach) for a given site but has a number of drawbacks as well. 1.) Geodetic observations have to be available for a certain period of time before a reliable regression coefficient can be determined; this period of time may be as large as several years. 2.) The regression coefficients cannot be extrapolated to a new site (for which no data exist); 4.) The regression coefficient has been observed to change with time and with observing technique; 4.) Regression coefficients at coastal sites are time dependent due to interannual changes in the regional weather pattern (H.-P. Plag, personal communication, 2002); 5.) The regression coefficient can only be used for vertical crustal motions; and 6.) It is uncertain that other pressure correlated geodetic signals are not being 'absorbed' into the regression coefficient determination. So while this approach would lower the scatter on a given geodetic time series the most, one would always be uncertain whether only atmospheric loading effects were being removed with the correlation coefficient.

In a hybrid method, regression coefficients determined from a geophysical model instead of geodetic observations could be used to operationally correct observed vertical position determinations from local air pressure alone. The vertical deformation caused by the change in pressure, in this case, can then be given in terms of a local pressure anomaly. The regression coefficients can be determined by fitting local pressure to the vertical deformation predicted by the geophysical model. Regression coefficients determined in this manner would still suffer from both the

uncertainty in the Green's function and the quality of the air pressure data.

In February 2002, the Special Bureau on Loading (SBL) was established within the IERS. The charge of the SBL is to promote, stimulate and coordinate work and progress towards a service providing information on Earth surface deformation due to surface mass loading, including the atmosphere, ocean and continental hydrosphere. In establishing the SBL the IERS is recommending that the convention for computing atmospheric loading corrections will be based on the geophysical model approach.

At the 2002 IERS Meeting in Munich, the IERS adopted the convention that corrections for surface load variations including the atmosphere should be determined using the geophysical model approach. Further, these corrections should be obtained from the IERS SBL. The point of this recommendation is to ensure that comparisons of geodetic time series between different observing techniques or within the same technique but at different times and locations have a consistent atmospheric pressure loading (and later also non-tidal ocean and continental hydrological loading) correction applied.

The ultimate goal of the SBL is to provide in near real-time a consistent global solution data set, describing at the surface, deformation due to all surface loads (including atmospheric pressure variations) in reference frames relevant for direct comparison with geodetic observing techniques. The SBL will provide global gridded solutions of 3-D displacements and time series of displacements for all IERS sites. Time series will be determined from 1985 to the present. Displacements will be determined for both the European Center for Medium Range Weather Forecasts and the National Center for Environmental Prediction operational pressure data sets for the inverted barometer and the non-inverted barometer ocean models. For more information see: <<sup>17</sup>>.

Regression coefficients based on a geophysical model are already available for a number of VLBI sites through the SBL web page and the IERS Convention's web page <<sup>18</sup>>. The regression coefficients were computed using 18 years of the NCEP Reanalysis Data (1 Jan. 1980 to 31 Dec. 1997). The data are 6 hourly values of surface pressure given on a 2.5°×2.5° global grid. Vertical crustal motions at a particular site are modeled by convolving Farrell's (1972) Greens functions for a Gutenberg-Bullen A Earth model. The ocean was assumed to be inverse barometric for the calculations. The regression results (mm/mbar) are determined via a linear regression between the modeled crustal displacements and the local surface pressure determined from the NCEP data set. An inverted barometer model was used in determining the ocean's response to pressure.

For more information on atmospheric pressure loading and geodetic time series, see the references listed in the extended bibliography.

## 7.2 Displacement of Reference Points of Instruments

### 7.2.1 VLBI Antenna Thermal Deformation

The following has been excerpted from the Explanatory Supplement to the IERS Conventions (1996) Chapters 6 and 7 (Schuh, 1999).

Most VLBI telescopes are of Cassegrain type with alt-azimuth or polar mount and secondary focus. Figures 7.1 and 7.2, based on Nothnagel *et al.* (1995), show the principles of these antenna mounts. The height of the concrete foundation is denoted by  $h_f$ , the height of the antenna

<sup>17</sup><http://www.gdiv.statkart.no/sbl>

<sup>18</sup><ftp://maia.usno.navy.mil/conv2000/chapter7/atmospheric.regr>

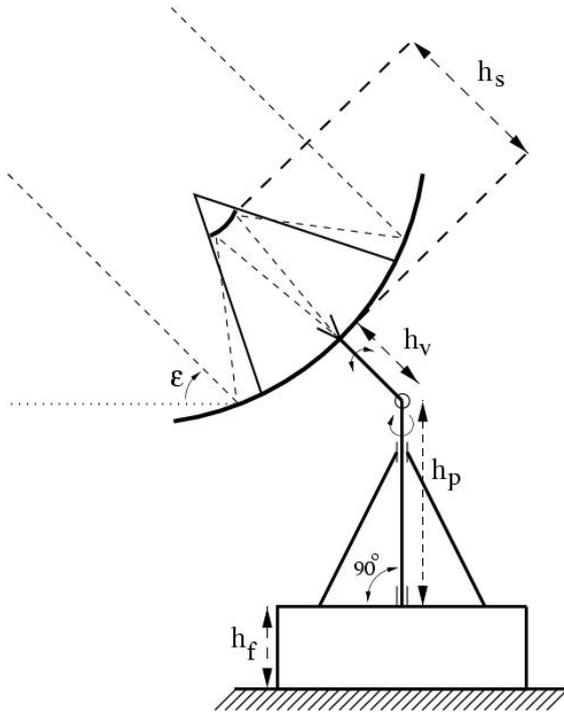


Fig. 7.1 Alt-azimuthal telescope mount.

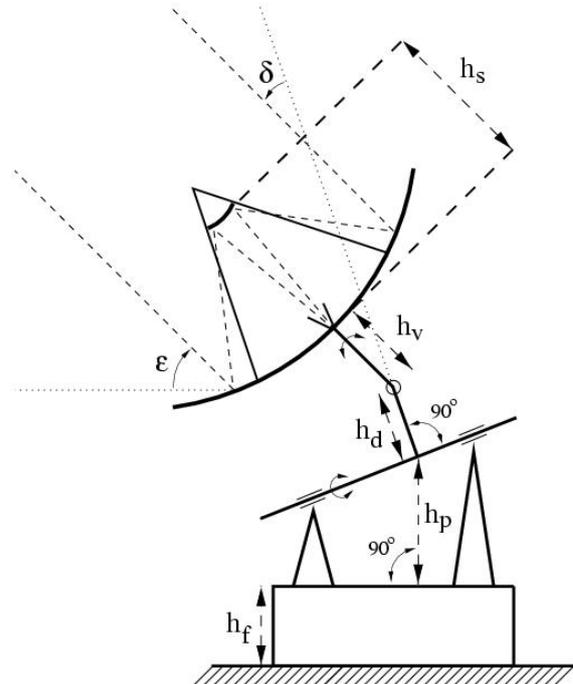


Fig. 7.2 Polar telescope mount.

pillar by  $h_p$ , the height of the vertex by  $h_v$ , the height of the subreflector by  $h_s$  and the declination shaft by  $h_d$ .

Then, the thermal deformation effect  $\Delta\tau$  in  $s$  on the VLBI delay measurement  $\tau$  can be modeled. For alt-azimuth mounts

$$\Delta\tau = \frac{1}{c} \cdot \left[ \gamma_f \cdot (T(t - \Delta t_f) - T_0) \cdot (h_f \cdot \sin(\varepsilon)) + \gamma_a \cdot (T(t - \Delta t_a) - T_0) \cdot (h_p \cdot \sin(\varepsilon) + h_v - 1.8 \cdot h_s) \right]. \quad (28)$$

For polar mounts

$$\Delta\tau = \frac{1}{c} \cdot \left[ \gamma_f \cdot (T(t - \Delta t_f) - T_0) \cdot (h_f \cdot \sin(\varepsilon)) + \gamma_a \cdot (T(t - \Delta t_a) - T_0) \cdot (h_p \cdot \sin(\varepsilon) + h_v - 1.8 \cdot h_s + h_d \cdot \cos(\delta)) \right]. \quad (29)$$

In the above equations (28) and (29)  $c$  in  $m/s$  is the speed of light,  $\gamma_f$  and  $\gamma_a$  in  $1/^\circ C$  are the expansion coefficients for the foundation and for the antenna, respectively, and  $h_f$ ,  $h_p$ ,  $h_v$ ,  $h_s$  and  $h_d$  are the dimensions of the telescopes in  $m$ . For prime focus antennas, the factor for  $h_s$  is 0.9 instead of 1.8. The temperature of the telescope structure is denoted by  $T$ , and  $T_0$  is a reference temperature, *e.g.*  $20^\circ C$  which is the usual reference temperature used when designing and constructing buildings. If the actual temperature of the telescope structure is not available, which might be the case at most VLBI sites, the surrounding air temperature can be taken instead. The time delay between the change in the surrounding air temperature and the expansion of the telescope structure is denoted by  $\Delta t_f$  for the foundation part and by  $\Delta t_a$  for the antenna part and depend strongly on the material of the telescope. Measurements yielded values of  $\Delta t_a=2$  hours for a steel telescope structure (Nothnagel *et al.*, 1995) and of  $\Delta t_f=6$  hours for a concrete telescope structure (Elgered and Carlsson, 1995). The elevation and declination of the observed

radio source are denoted by  $\varepsilon$  and  $\delta$ . Table 7.6 contains the dimensions of some frequently used geodetic VLBI antennas and mean expansion coefficients.

Table 7.7 gives the thermal variation  $\Delta\tau$  of the VLBI delay observable, based on the telescope dimensions and expansion coefficients given in Table 7.6 and equations (28) and (29). Temperature variation ( $T - T_0$ ) of  $10^\circ\text{C}$  and radio source elevations between  $5^\circ$  and  $90^\circ$  were entered, time lags  $\Delta t_f$  and  $\Delta t_a$  were assumed to be zero.

For big VLBI telescopes, variations in the VLBI delay observations of several picoseconds can occur. Regarding a baseline of two telescopes with the signal from the radio source arriving first at site 1, the total effect on the measured delay on the baseline is:

$$\Delta\tau_{\text{baseline}} = \Delta\tau_1 - \Delta\tau_2.$$

Table 7.6 Dimensions and expansion coefficients of frequently used geodetic VLBI telescopes.

Telescope	Foundation part (concrete)		Antenna part (steel)				
	$h_f$	$\gamma_f$	$h_p$	$h_v$	$h_s$	$h_d$	$\gamma_a$
	m	$1/^\circ\text{C}$	m	m	m	m	$1/^\circ\text{C}$
Effelsberg	0.0	$1.0 \times 10^{-5}$	50.0	8.5	28.0	—	$1.2 \times 10^{-5}$
Hartebeesthoek	0.0	$1.0 \times 10^{-5}$	12.7	2.3	9.4	6.7	$1.2 \times 10^{-5}$
Madrid	3.0	$1.0 \times 10^{-5}$	16.8	2.7	10.8	—	$1.2 \times 10^{-5}$
Matera	3.0	$1.0 \times 10^{-5}$	10.5	3.8	5.7	—	$1.2 \times 10^{-5}$
Medicina	2.3	$1.0 \times 10^{-5}$	15.5	4.3	4.3	—	$1.2 \times 10^{-5}$
Noto	2.2	$1.0 \times 10^{-5}$	15.7	4.2	5.0	—	$1.2 \times 10^{-5}$
O'Higgins	1.0	$1.0 \times 10^{-5}$	6.2	—	—	—	$1.2 \times 10^{-5}$
Onsala	11.3	$1.0 \times 10^{-5}$	2.9	3.4	5.5	—	$1.2 \times 10^{-5}$
Westford	16.9	$1.0 \times 10^{-5}$	2.0	3.0	3.6	—	$1.2 \times 10^{-5}$
Wettzell	8.0	$1.0 \times 10^{-5}$	4.0	3.7	7.9	—	$1.2 \times 10^{-5}$

Table 7.7 Thermal variations  $\Delta\tau$  in *ps* of the VLBI delay observable for frequently used geodetic VLBI telescopes for a temperature variation of  $10^\circ\text{C}$  and different radio source elevations.

Telescope	$(T - T_0) = 10^\circ\text{C}$			
	Elevation $\varepsilon$			
	$5^\circ$	$30^\circ$	$60^\circ$	$90^\circ$
	$\Delta\tau$	$\Delta\tau$	$\Delta\tau$	$\Delta\tau$
	<i>ps</i>	<i>ps</i>	<i>ps</i>	<i>ps</i>
Effelsberg	-15.0	-6.8	+0.6	+3.2
Hartebeesthoek	-7.8	-5.0	-1.9	+0.1
Madrid	-6.0	-2.8	0.0	+1.0
Matera	-2.1	0.0	+1.9	+2.6
Medicina	-0.8	+2.1	+4.6	+5.6
Noto	-1.3	+1.6	+4.2	+5.1
O'Higgins	0.2	+1.4	+2.4	+2.8
Onsala	-2.2	-0.1	+1.7	+2.3
Westford	-0.8	+1.8	+4.2	+5.0
Wettzell	-3.8	-2.0	-0.5	0.0

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